Using Non-Linear Mixed Models for Agricultural Data

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Outline

Objectives of Statistical Modeling

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1. Develop the simplest model which still captures the structure of the data
2. Interpret the model (give meaning to the parameters)
3. Generate predictions (validation)

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Non-Linear and Mixed Models

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1. Parsimony
2. Interpretability
3. Model the mean structure

Mixed Models

1. Flexibility
2. Hierarchy
3. Model the error structure

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Barley N response trials

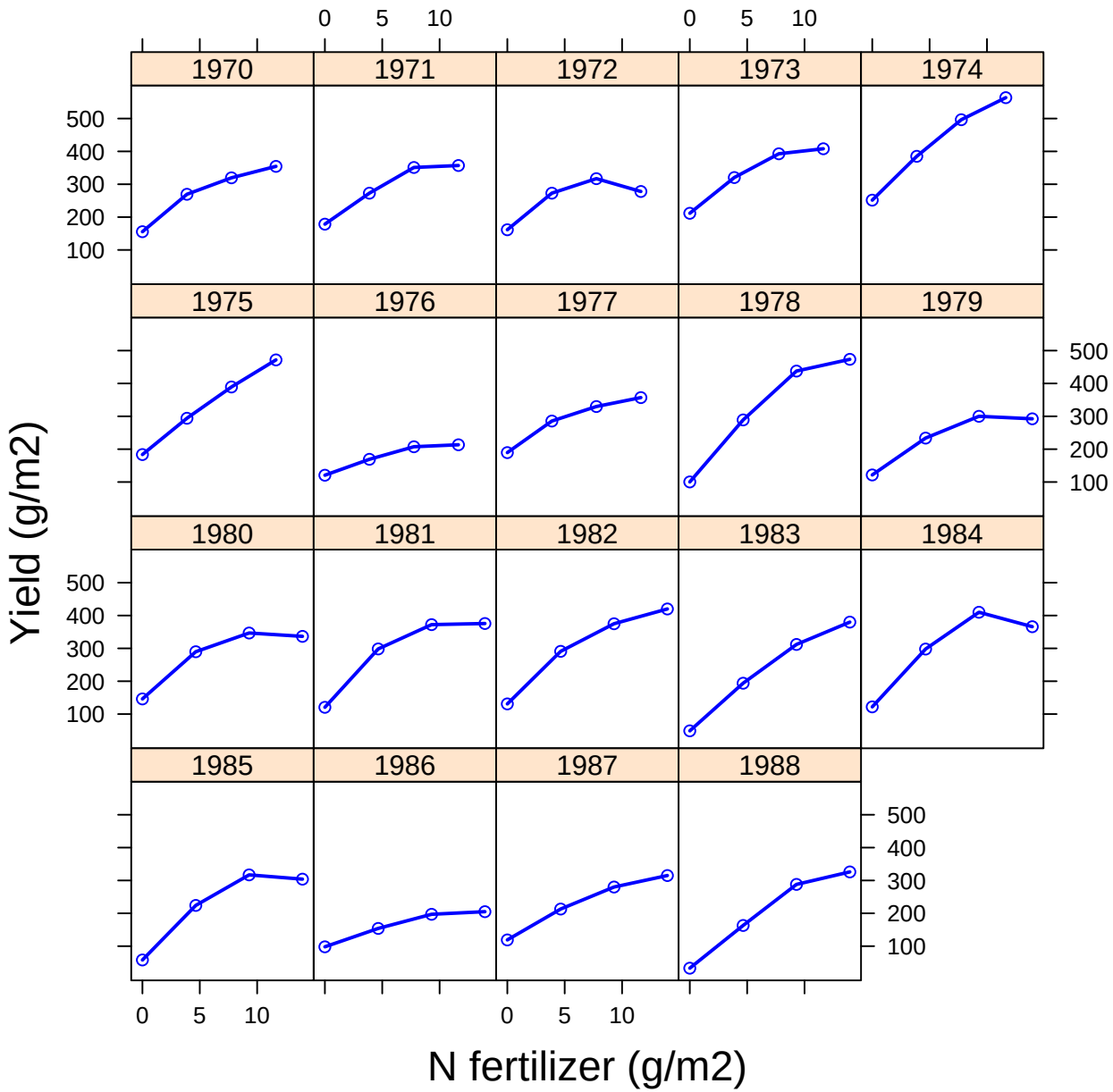
Aril Vold (1998). A generalization of ordinary yield response functions. Ecological Applications. 108:227-236.

Details

- 19 years of data, Norway
- N rates (0, 3.38, 7.76 and 11.69 g N m^{-2}) raised by 20% in 1978

Agronomic Questions

1. How does it respond to N?
2. How does it vary among years?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Yield (g/m2)
N fertilizer (g/m2)
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Basics of Statistical Models

$$y = f(x, \theta) + \epsilon$$

where,

$y =$ observed

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$$\mathcal{D} = \mathcal{M} + \mathcal{E}$$

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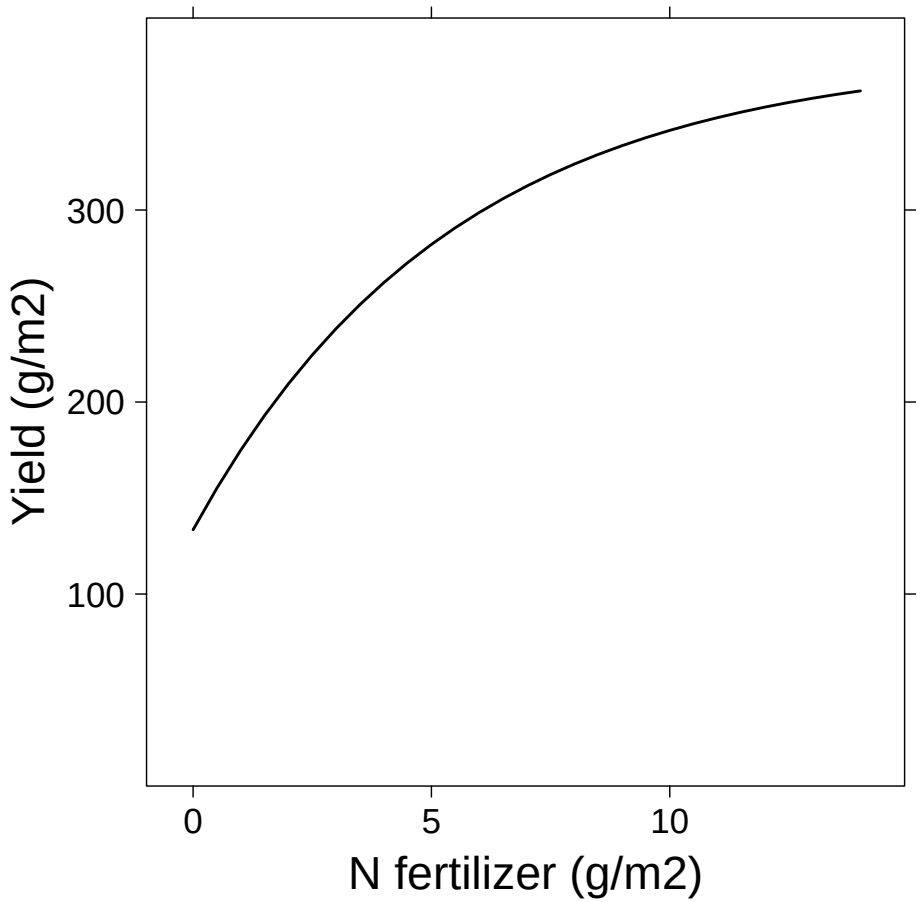
Choosing the Mean Structure

Asymptotic Regression Model

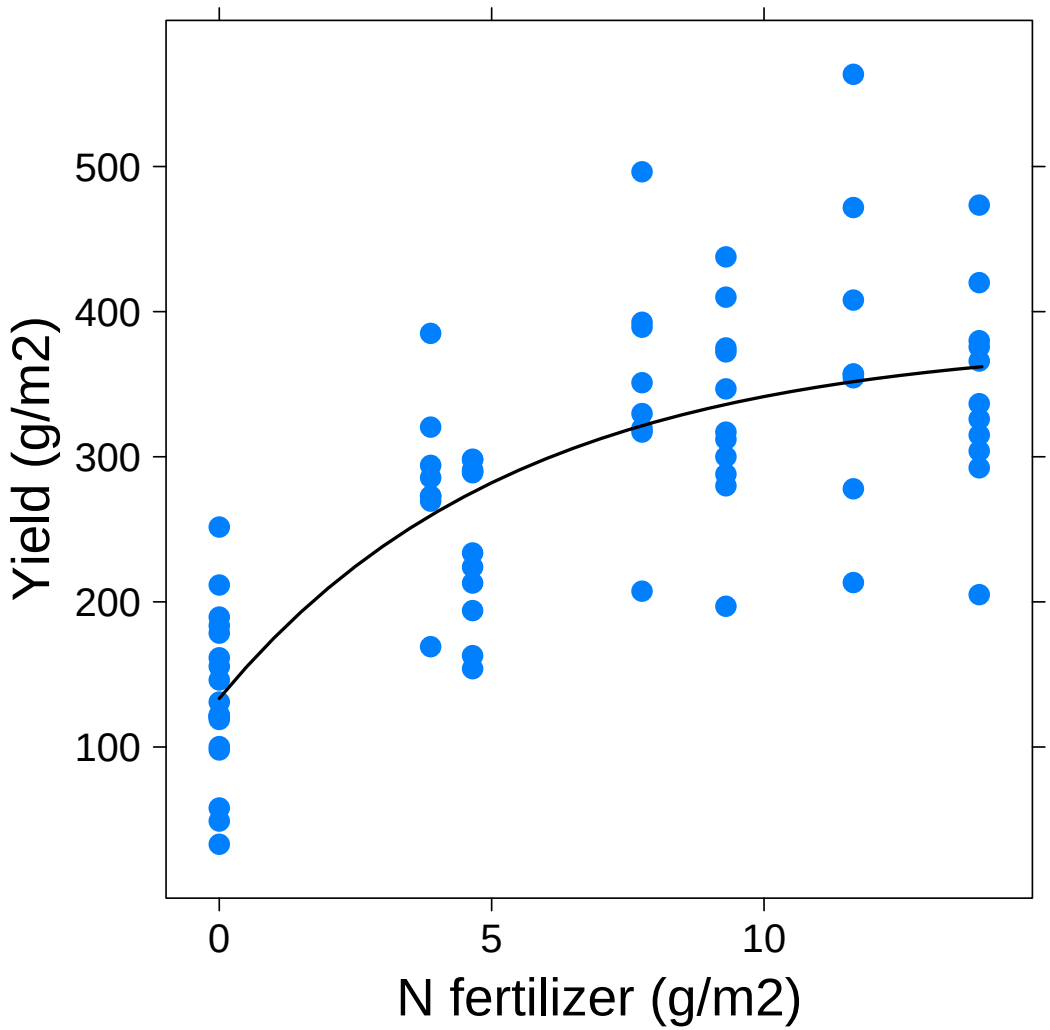
$$y = \theta_1 + (\theta_2 - \theta_1) \times \exp(-\exp(\theta_3) \times x)$$

where,
θ_1 is the maximum value of y
θ_2 is the value of y for $x = 0$.
θ_3 is the growth rate of y



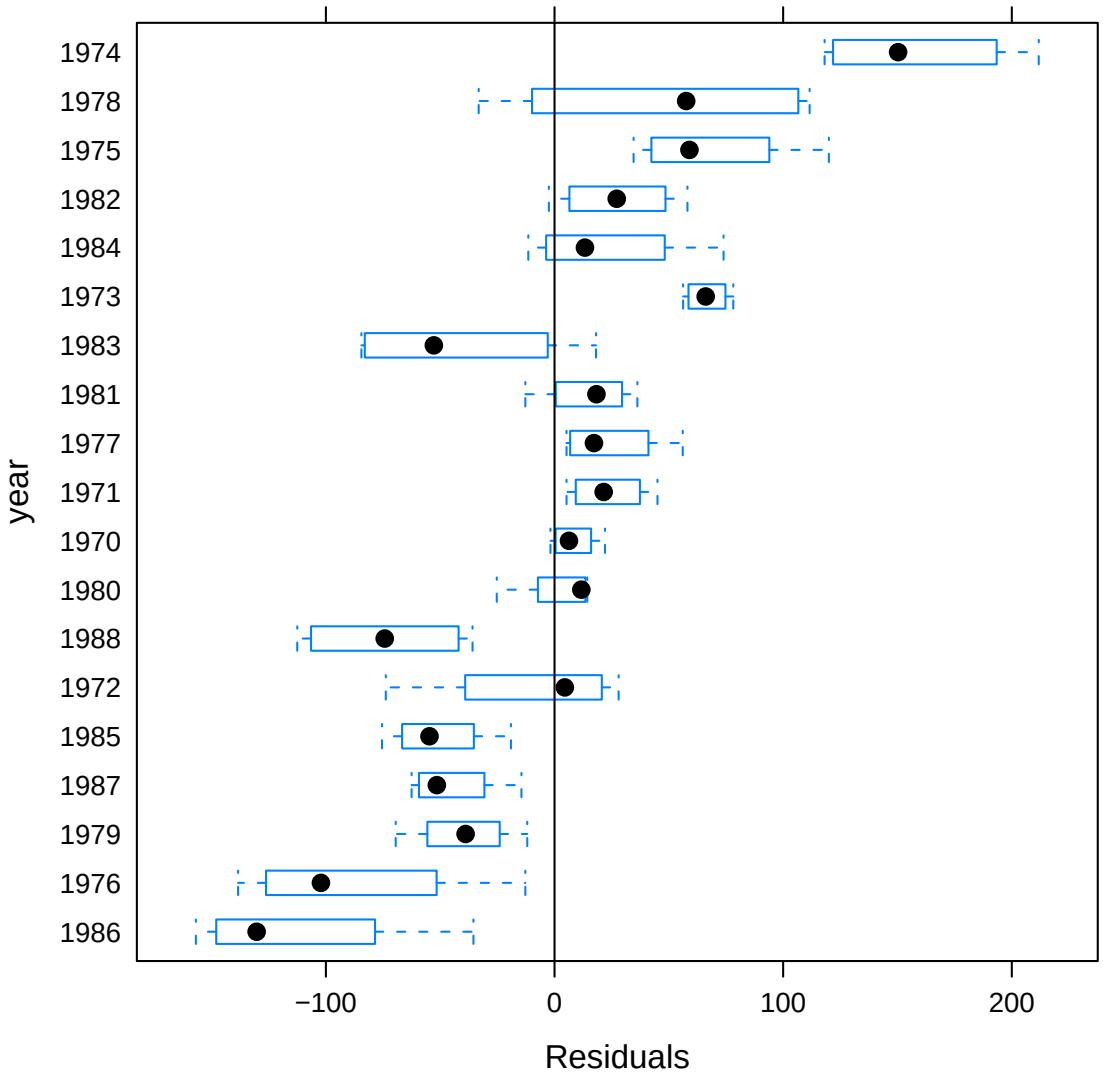
Barley N response trials

Non-linear regression with years combined

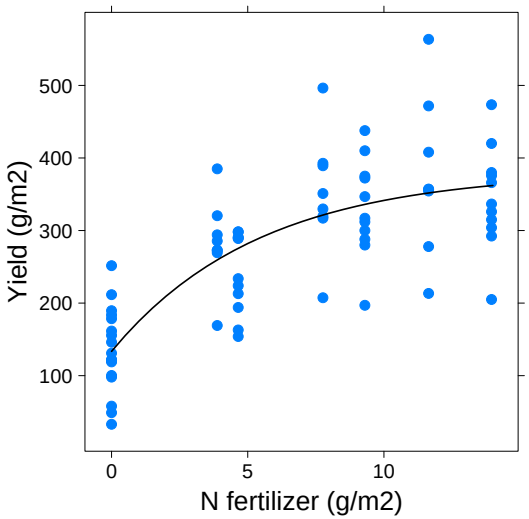


Barley N response trials

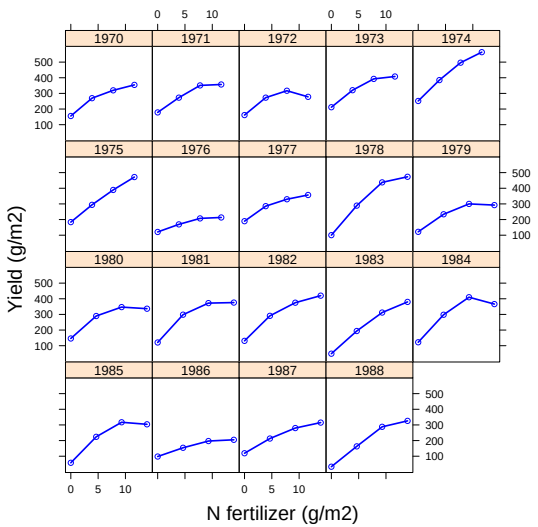
Box-plots of residuals for each year



Barley N response trials



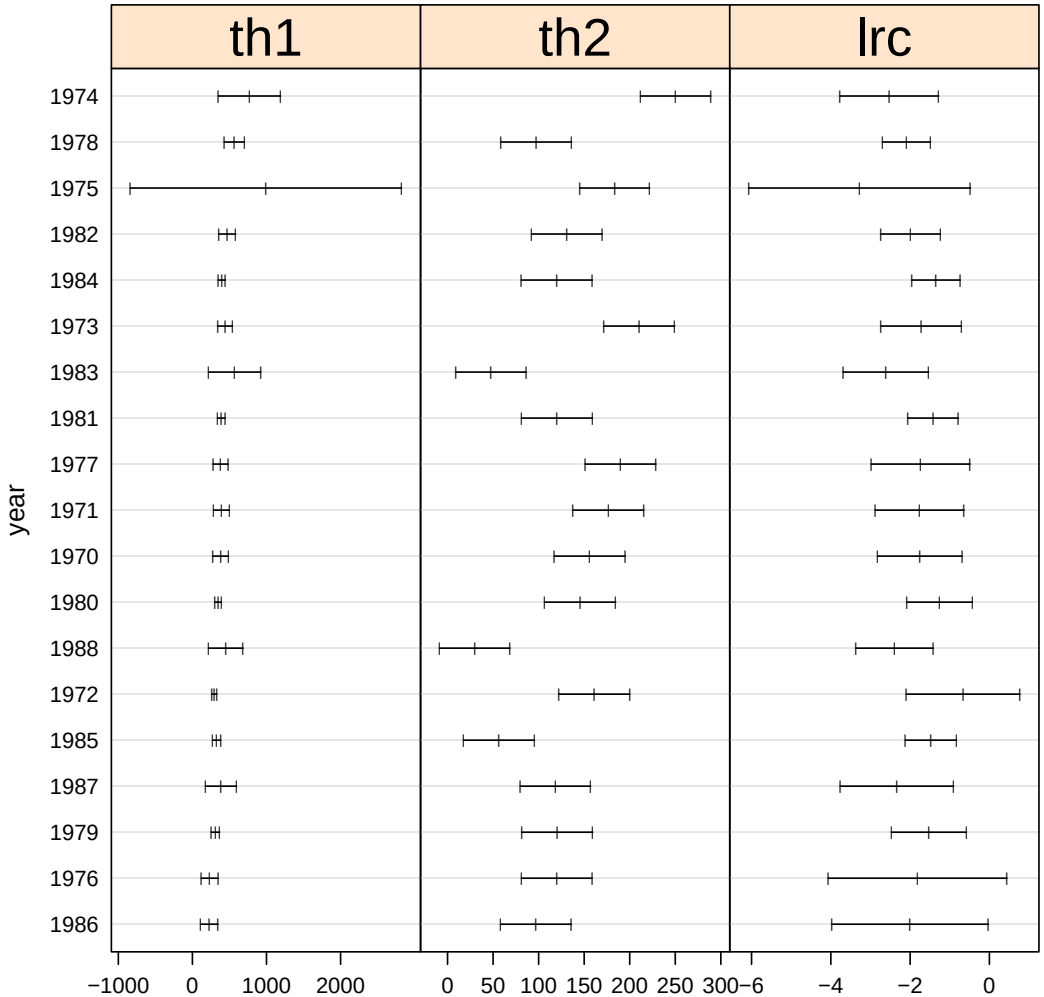
- One single regression to all the data
  - Wide confidence intervals
  - Ignores the structure of the data

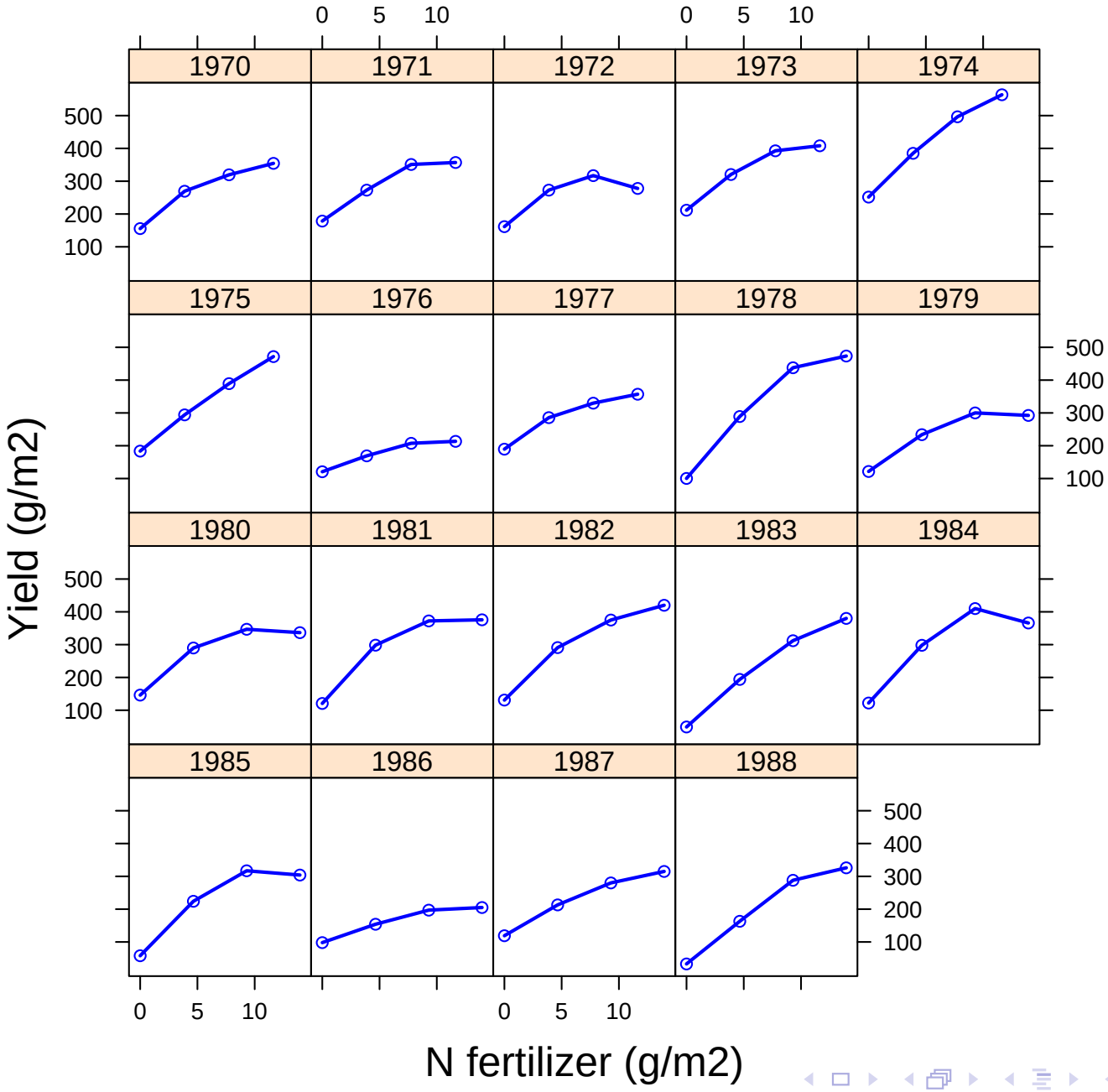


- Fitting one function for each separate year
  - Over-parameterized model
  - 3 parms $\times$ 19 y $=$ 57 parms

Barley N response trials

Confidence Intervals for Non-linear regressions for each year

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Non-Linear Mixed Model

Asymptotic regression with random effects

$$y_{ij} = (\theta_1 + b_{1i}) + ((\theta_2 + b_{2i}) - (\theta_1 + b_{1i})) \times \exp(-\exp(\theta_3 + b_{3i}) \times x_{ij}) + \epsilon_{ij}$$

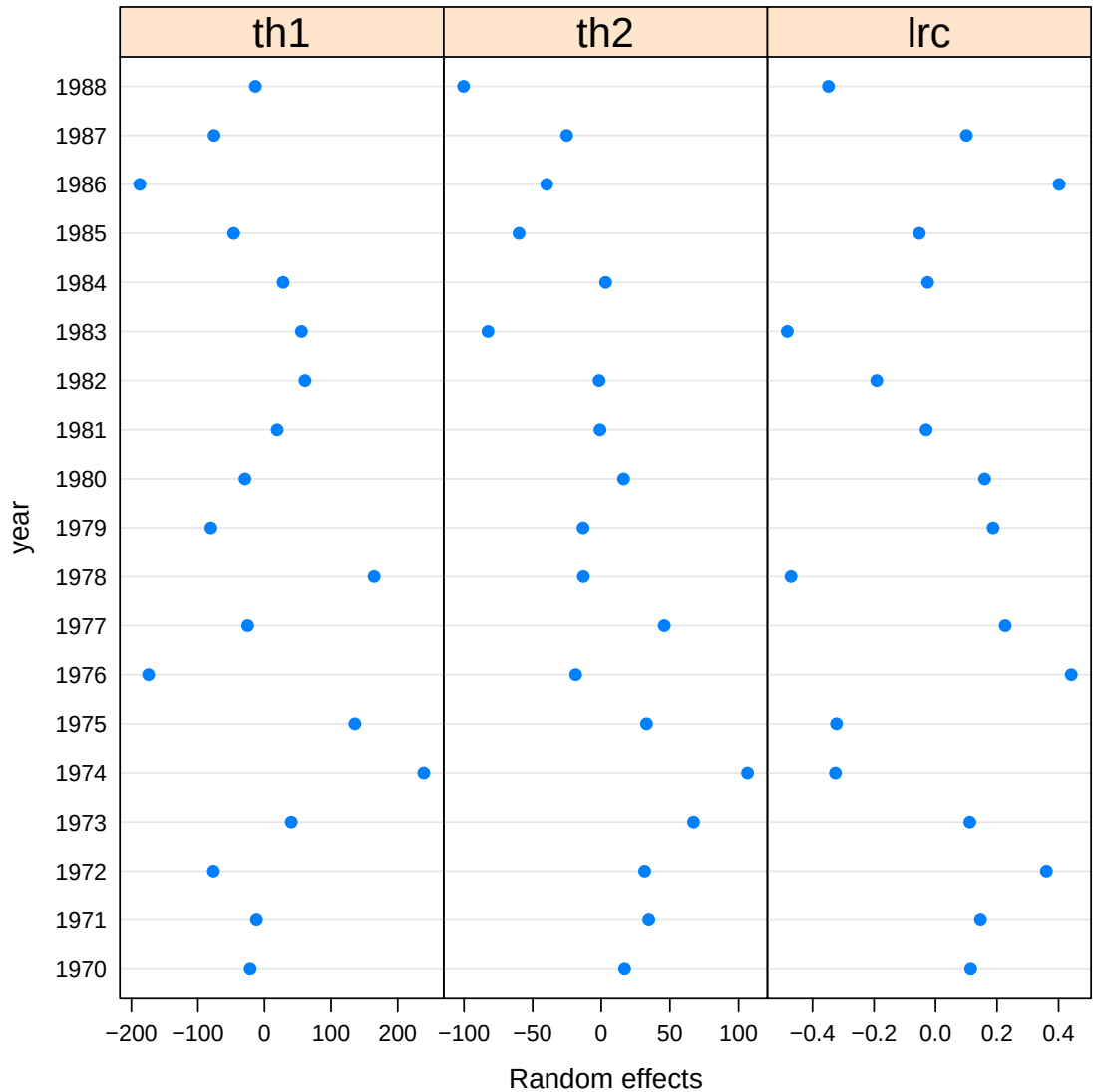
$i =$ the year (or experimental unit)
$j =$ the N rate

$$\boldsymbol{b_i} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi}), \quad \epsilon_{ij} \sim \mathcal{N}(0, \sigma^2)$$

$$\boldsymbol{\Psi} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

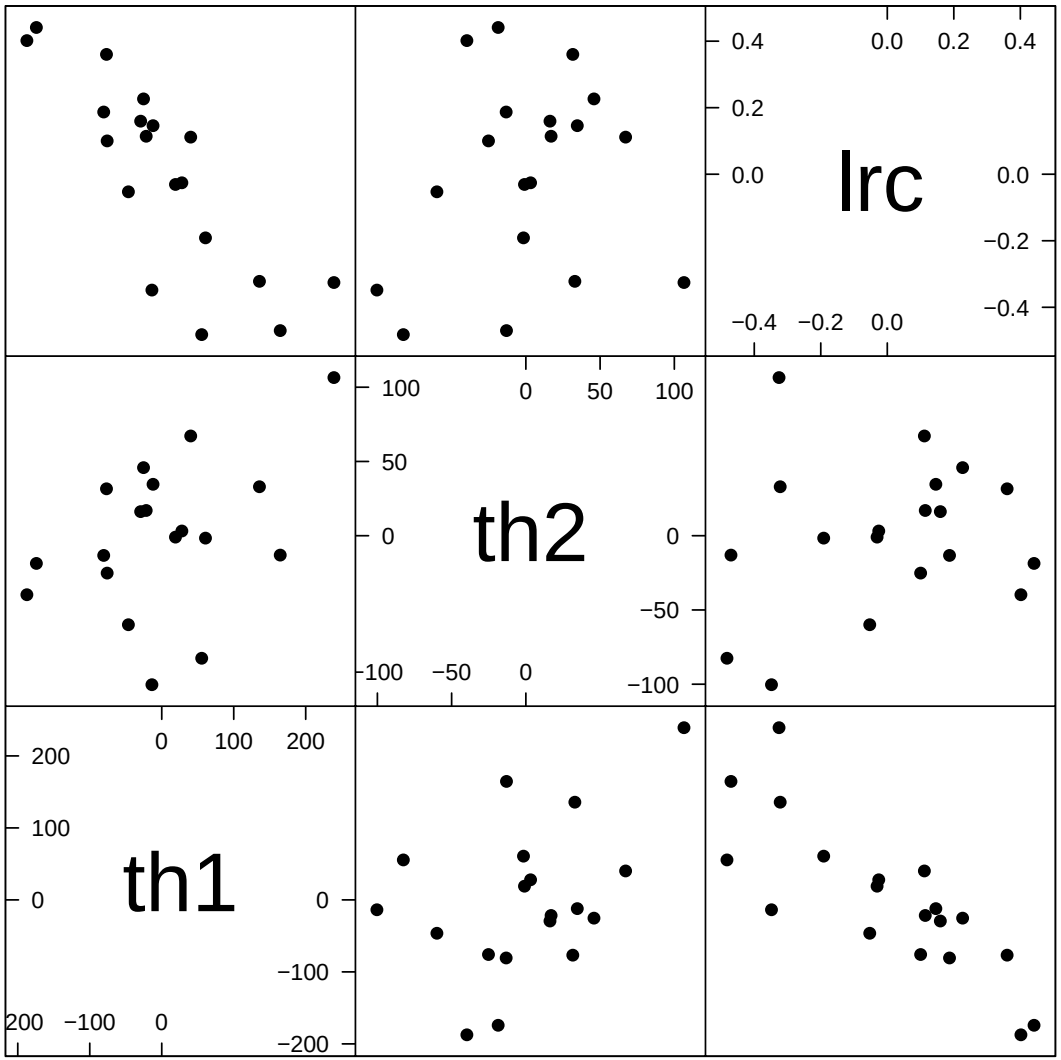
Random Effects

Dot plot for the random effects



Random Effects

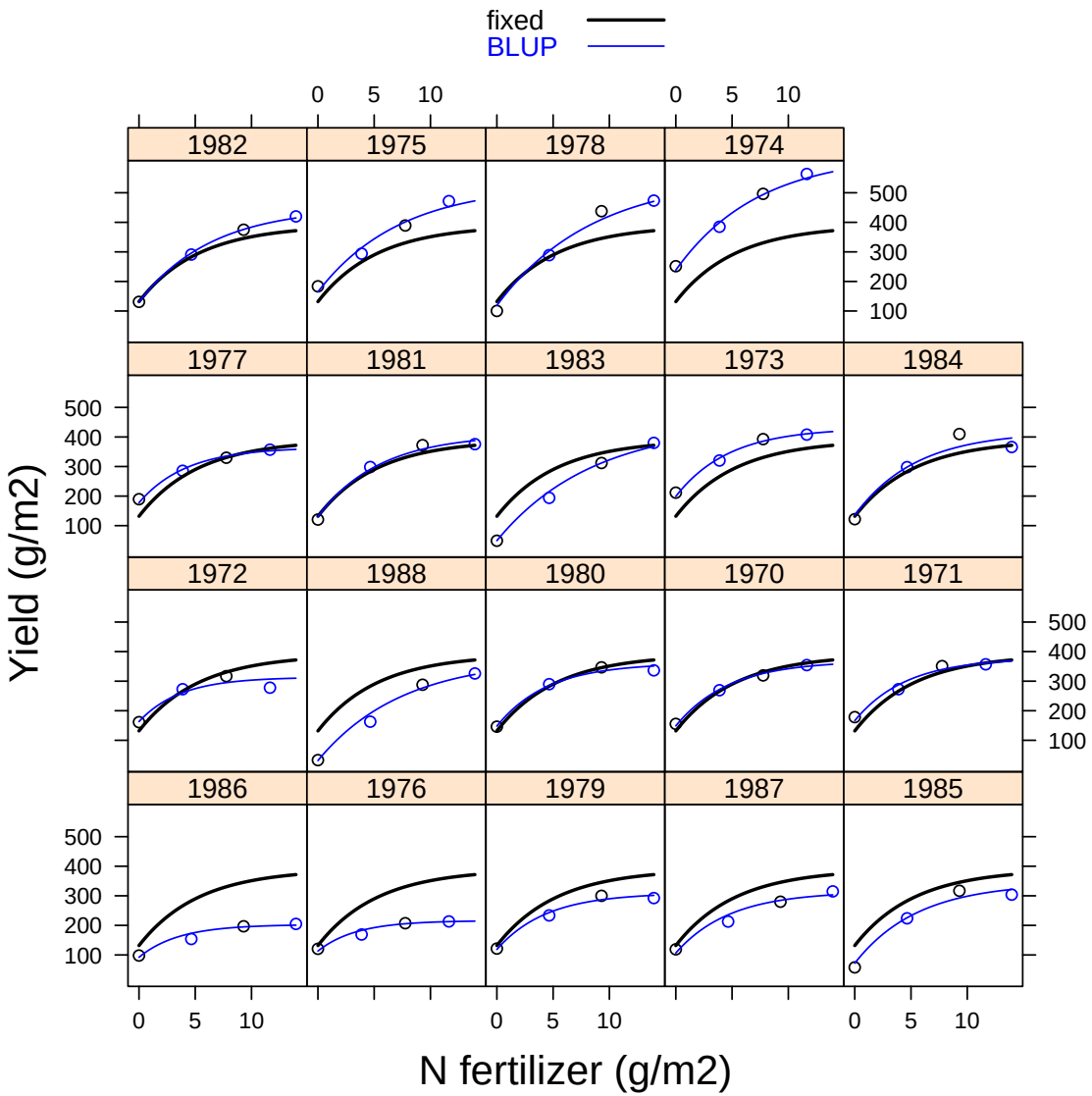
Scatter plot matrix for the random effects



Scatter Plot Matrix

Non-Linear Mixed Model

Fixed and BLUP



Comparison of NLS and NLME

Estimate, and 95% confidence intervals for the three parameters of the asymptotic regression model (NLS) and the mixed-effects model (NLME).

Fixed term	**Estimate**	**Lower**	**Upper**
θ_1 NLS	381	335	507
θ_1 NLME	390	337	443
θ_2 NLS	133	101	166
θ_2 NLME	132	107	157
lrc NLS	-1.7	-2.7	-1.1
lrc NLME	-1.7	-1.9	-1.4
$\hat{\sigma}$ NLS	71.2		
$\hat{\sigma}$ NLME	18.8	13.8	25.6

Summary: Using NLME

- NLME are able to accomodate the mean and error structure
- NLME produce a parsimonious and easy to interpret model
- The NLME estimates are more accurate and the confidence intervals are narrower

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Application to Meta-analysis

Meta-analysis of the effects of management factors on Miscanthus x giganteus growth and biomass production. Miguez et al (2008) Agricultural and Forest Meteorology. 148:1280-1292.

R Code and Data

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Questions?