Growth functions

AGRON 590 MG: Crop-Soil Modeling

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Modeling Leaf-level photosynthesis

Example of a non-linear function: Rectangular hyperbola

$$A = \frac{\phi Q + A_{sat} - \sqrt{(\phi Q + A_{sat})^2 - 4\theta\phi Q A_{sat}}}{2\theta} - R_d$$

where

ϕ = apparent quantum efficiency

Q = Quantum flux

A_{sat} = saturated rate of photosynthesis

θ = curvature

R_d = dark respiration

General function

$$y = f(x)$$

where

y is the response

f is an undefined function
x is the explanatory variable

A simple function
Simple Linear Model

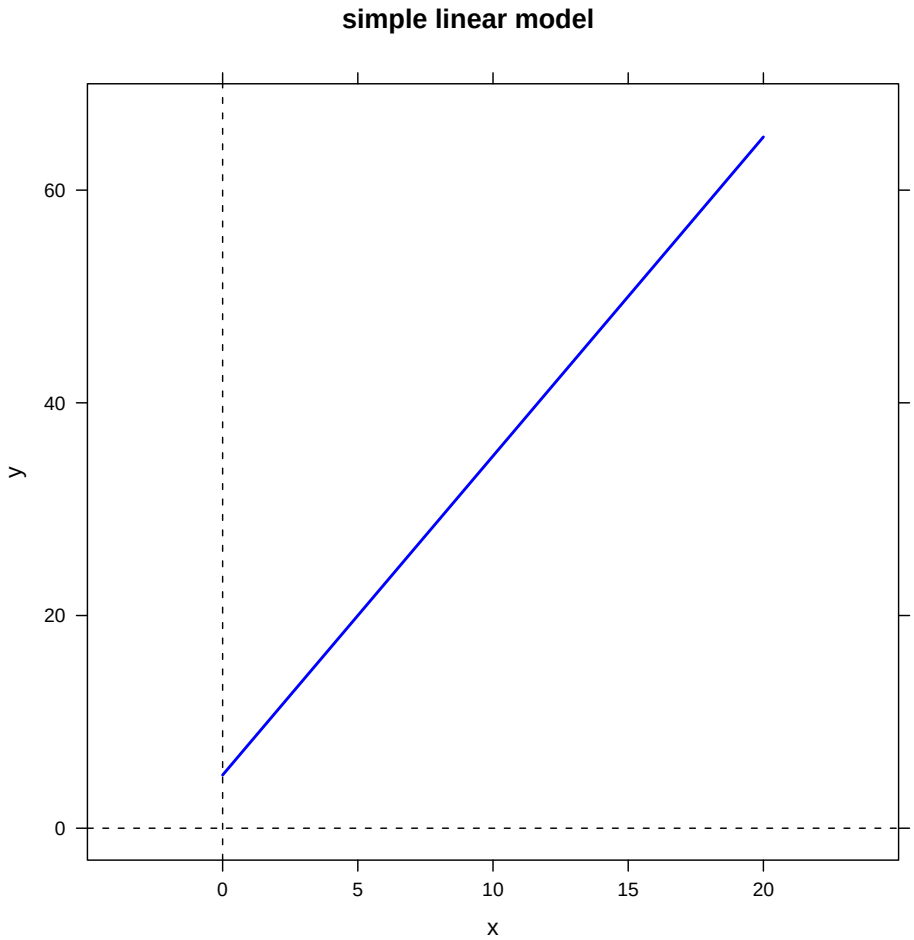
$$y = f(x)$$

$$y = a + b \times x$$

$$a = 5$$

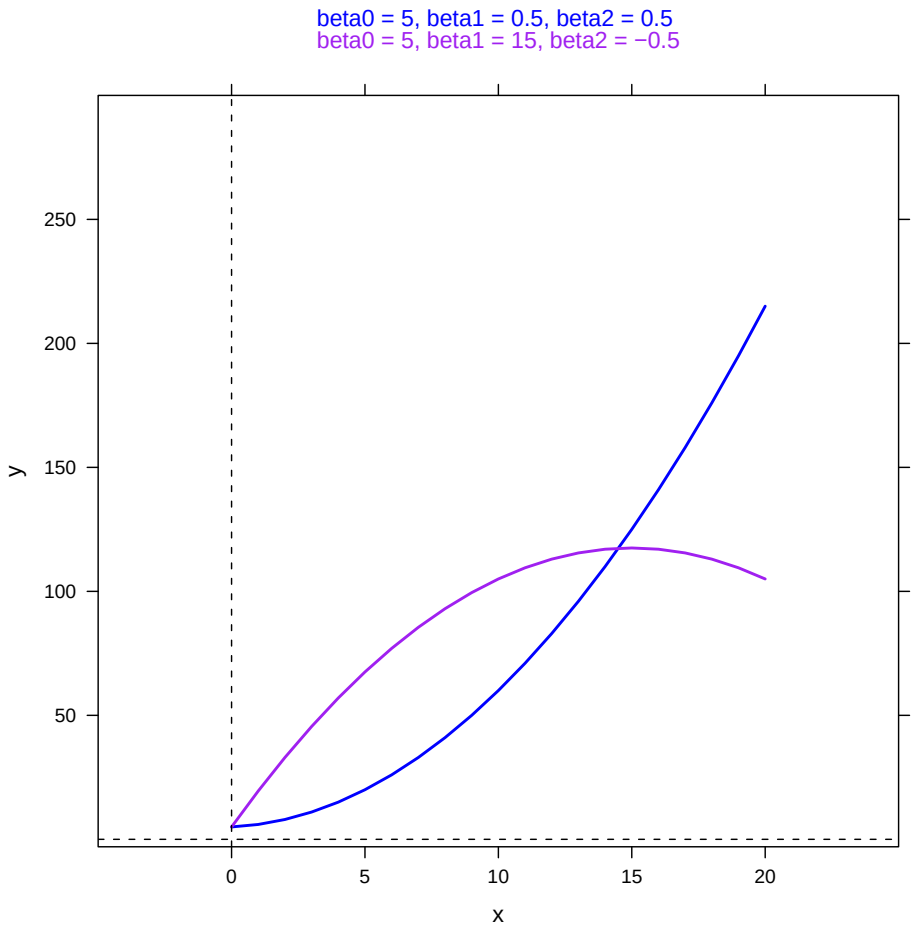
$$b = 3$$

$$x \quad [0, 20]$$



Extending the linear model

$$y = \beta_0 + \beta_1 x + \beta_2 x^2$$



A simple function and its rate of change

$$W = f(t)$$

$$\frac{dW}{dt}$$

Example: exponential growth

$$\frac{dW}{dt} = \mu W$$

How do we integrate this?

Example: exponential growth

$$\frac{dW}{dt} = \mu\, W$$

Separate variables

Example: exponential growth

$$\frac{dW}{dt} = \mu\, W$$

$$\frac{1}{W} dW = \mu\, dt$$

Example: exponential growth

$$\frac{dW}{dt} = \mu\, W$$

$$\frac{1}{W} dW = \mu\, dt$$

$$\int \frac{1}{W} dW = \int \mu\, dt$$

Example: exponential growth

$$\frac{dW}{dt} = \mu\, W$$

$$\frac{1}{W} dW = \mu\, dt$$

$$\int \frac{1}{W} dW = \int \mu\, dt$$

$$\ln X = \mu t$$

Example: exponential growth

$$\frac{dW}{dt} = \mu\, W$$

$$\frac{1}{W} dW = \mu\, dt$$

$$\int \frac{1}{W} dW = \int \mu\, dt$$

$$\ln W = \mu\, t$$

$$\exp(\ln W) = \exp(\mu\, t)$$

Example: exponential growth

$$\frac{dW}{dt} = \mu\, W$$

$$\frac{1}{W} dW = \mu\, dt$$

$$\int \frac{1}{W} dW = \int \mu\, dt$$

$$\ln W = \mu\, t$$

$$W = \exp(\mu\, t)$$

Example: exponential growth

$$\frac{dW}{dt} = \mu\, W$$

$$\frac{1}{W} dW = \mu\, dt$$

$$\int \frac{1}{W} dW = \int \mu\, dt$$

$$\ln W = \mu\, t$$

$$W = \exp(\mu\, t)$$

When $t = 0$, $W = 1$ but we can add a parameter for the initial condition

$$W = W_0\ \exp(\mu\, t)$$

Tools in R
For linear models

Go to R for linear models

Tools in R

For non-linear models

nls selfStart nlme

Example in R
For non-linear models

Go to R

Logistic function

$$y = \frac{A}{1 + \exp((x_{mid} - x)/scal)}$$

$A =$ asymptote
$x_{mid} =$ midpoint
$scal =$ 'scale' or spread